

Exercise 47

If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.

Solution

Plug in $x = 3$ to $g(x)$.

$$g(3) = 3f(3) = 3(4) = 12$$

The point at which the line intersects $g(x)$ is then $(3, 12)$. What's needed now is a slope. Take the derivative of $g(x)$ using the product rule.

$$\begin{aligned}g'(x) &= \frac{d}{dx}[xf(x)] = \left[\frac{d}{dx}(x) \right] f(x) + xf'(x) \\ &= (1)f(x) + xf'(x) \\ &= f(x) + xf'(x)\end{aligned}$$

Evaluate it at $x = 3$.

$$g'(3) = f(3) + 3f'(3) = 4 + 3(-2) = -2$$

The equation of the tangent line to the graph of g at the point where $x = 3$ is therefore

$$y - 12 = -2(x - 3).$$